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Compressible magnetoconvection in oblique fields: linearized theory and simple nonlinear models

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The linear stability of a layer of compressible fluid, permeated by an oblique magnetic field, is discussed. It is shown that regardless of the system parameters, all bifurcations generically lead to travelling waves. Wave speeds and direction of the wave propagation are investigated. Symmetry arguments are used to show that when the field is almost vertical, waves with a wave vector aligned with the tilt are preferred over those with a wave vector perpendicular to the tilt. The nonlinear development of the travelling waves is explored using simple model equations.

1. Introduction

There have been numerous studies undertaken of the nonlinear interaction of magnetic fields and compressible convection in a horizontal fluid layer (Nordlund & Stein 1989; Cattaneo 1984; Hurlburt & Toomre 1988; Hurlburt *et al.* 1989; Weiss *et al.* 1990). These papers were all motivated by the desire to understand the physics of sunspots and flux tubes in the solar convection zone. In all of them the magnetic field imposed at the start of the calculation was uniform and vertical and during the subsequent evolution the field was assumed to remain vertical at the horizontal boundaries of the computational domain. In these circumstances it is possible to find both steady and oscillatory motions near the onset of instability; and the oscillations can appear either as standing or travelling waves. A considerable amount of effort has been expended by the above authors in understanding the interaction of these various types of solutions. Similar remarks apply to convection in an imposed horizontal field. This has been less intensively studied and then primarily in the Boussinesq limit (Proctor & Weiss 1982; Arter 1983; Knobloch 1986; Brownjohn *et al.* 1992).

However, it is clear that in general solar magnetic fields are neither parallel nor perpendicular to the gravity vector. In the Boussinesq limit, for which the material properties are constant and there is a symmetry between the top and bottom of the layer, the bifurcation structure is unaffected when the imposed field is oblique. In the compressible case the symmetry disappears and it is then impossible to find bifurcations leading to steady convection except for special values of parameters. Now all solutions arising from initial bifurcations take the form of travelling waves; and the waves have different stability properties depending upon the direction of their phase velocity with respect to the direction of the imposed magnetic field. Standing waves can no longer appear at primary bifurcations and indeed their

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analogues in the oblique case (which are actually modulated waves) can only arise through secondary bifurcations from travelling waves.

It is clearly of interest to know the preferred direction of the waves and their phase speeds as functions of the angle of obliquity and strength of the imposed field. The full resolution of these questions will have to await a proper nonlinear analysis of the governing equations. In this paper we solve the linear stability problem for various values of the parameters to allow us to deduce the small-amplitude behaviour. We then indicate by means of simple nonlinear model equations how the various solution branches may interact in the nonlinear regime.

The plan of the paper is as follows. In §2 we present the equations and boundary conditions of our model. Section 3 gives the results of linearized theory, and §4 discusses the extension to three dimensions. Section 5 details travelling wave solutions for our simple nonlinear model equations. In §6 we conclude and discuss the relevance of our results to convection in the vicinity of a sunspot.

2. Equations

We shall analyse compressible convection in a stratified atmosphere identical to that considered in Hurlburt *et al.* (1989), to which the reader is referred for full details of the basic state and non-dimensionalizaton. The only difference from the situation considered there is that now the imposed field is not vertical but tilted clockwise by an angle ϕ, $0 \leqslant \phi \leqslant \frac{1}{2}\pi$ (see figure 1). We assume this atmosphere experiences a uniform gravitational acceleration g directed downwards and possesses a constant shear viscosity μ, a constant thermal conductivity K, a constant magnetic diffusivity η, and a constant magnetic permeability μ_0. Further, we shall assume that the fluid satisfies the equation of state for a perfect monatomic gas with constant heat capacities c_v and c_p.

We assume that there is no dependence on the horizontal component y so that the flow and the field are two-dimensional depending only on the coordinates x (horizontal) and z (vertical, increasing downwards). For the purposes of this paper we require only the linearized equations. The perturbation pressure, density, velocity, temperature, and magnetic field are denoted by P, ρ, $\boldsymbol{u}$, T and $\boldsymbol{B}$; we denote the static state by the subscript s. We then have the continuity equation

$$\partial\rho/\partial t + \rho_s \nabla\cdot\boldsymbol{u} + \boldsymbol{u}\cdot\nabla\rho_s = 0, \tag{2.1}$$

the compressible Navier–Stokes equations

$$\rho_s \frac{\partial \boldsymbol{u}}{\partial t} = -\nabla P + \rho g \hat{\boldsymbol{z}} + \frac{1}{\mu_0}(-\nabla(\boldsymbol{B}_s\cdot\boldsymbol{B}) + \boldsymbol{B}_s\cdot\nabla\boldsymbol{B}) + \mu(\nabla^2\boldsymbol{u} + \tfrac{1}{3}\nabla\nabla\cdot\boldsymbol{u}), \tag{2.2}$$

the total energy equation

$$\rho_s c_v\left(\frac{\partial T}{\partial t} + \boldsymbol{u}\cdot\nabla T_s\right) + P_s \nabla\cdot\boldsymbol{u} = K\nabla^2 T, \tag{2.3}$$

and the induction equation

$$\frac{\partial \boldsymbol{B}}{\partial t} + \boldsymbol{B}_s \nabla\cdot\boldsymbol{u} - \boldsymbol{B}_s\cdot\nabla\boldsymbol{u} = \eta\nabla^2\boldsymbol{B}. \tag{2.4}$$

These are augmented by the equation of state for a perfect gas which implies

$$P = R_*(\rho_s T + \rho T_s), \tag{2.5}$$

where R_* is the gas constant.

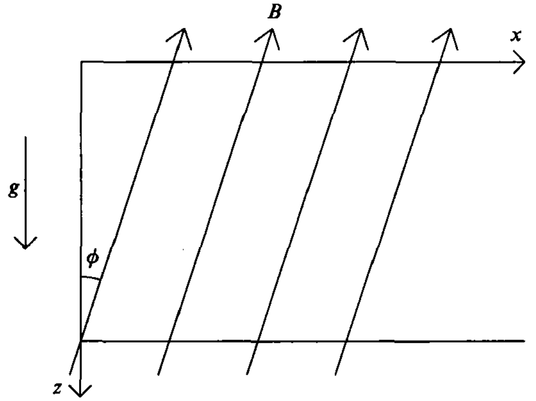

FIGURE 1. The model under consideration, showing the sense of the direction of slope of the magnetic field.

It is convenient to display all variables in dimensionless form hereafter. We shall take our unit of length to be the depth of the layer d. We will scale the density by the initial density ρ_0 (prior to the onset of convection) at the top of the unstable layer and the temperature by the (fixed) temperature difference ΔT across the unstable layer. Our time unit is $d/(R_* \Delta T)^{\frac{1}{2}}$ and is related to the sound travel time across the layer. The magnetic field is scaled by $\boldsymbol{B}_s$, the strength of the initially uniform field.

The stratification in the absence of motion when K is constant is simply that of a polytrope, where temperature, density and pressure have the form

$$T_s = z, \quad \rho_s = (z/z_0)^m, \quad P_s = z^{m+1}/z_0^m, \tag{2.6}$$

where the polytropic index m is determined by $m = gd/R_* \Delta T - 1$ and z_0 is an integration constant which represents the dimensionless temperature at the top of the layer. The dimensionless depth variable z ranges from $z = z_0$ to $z_0 + 1$, so the density contrast measuring the ratio of the density at the bottom to that at the top of the layer is

$$\chi = \left(\frac{z_0+1}{z_0}\right)^m. \tag{2.7}$$

The degree of instability may be measured by the Rayleigh number, which has a local value of

$$R = (m+1)(1-(m+1)(\gamma-1)/\gamma)\frac{z^{2m-1}}{\sigma \bar{K}^2 z_0^{2m}}, \tag{2.8}$$

where
$$\bar{K} = \frac{K}{(R_* \Delta T)^{\frac{1}{2}} \rho_0 d c_p} \tag{2.9}$$

is the dimensionless thermal conductivity,

$$\sigma = \mu c_p/K \tag{2.10}$$

is the Prandtl number and $\gamma = c_p/c_v$. Since R varies with depth, it is most convenient to evaluate it at midlayer, thereby setting $R(z_0+\frac{1}{2}) \equiv R_a$, since such a choice approaches the Boussinesq definition as $\chi \to 1$.

The effect of the magnetic field upon the convective stability can be measured by the Chandrasekhar number

$$Q = \frac{B_s^2 d^2}{\mu_0 \mu \eta}, \tag{2.11}$$

and the magnetic Prandtl number is defined as

$$\zeta_0 = \eta \rho_0 c_p / K. \tag{2.12}$$

The subscript zero, as used in z_0, ρ_0 and ζ_0, indicates evaluation of these variables at the top of the layer.

We assume that the temperature is fixed on the top and bottom horizontal surfaces. The field is constrained to make an angle ϕ with the vertical at these surfaces at all times. The vertical velocity and the horizontal viscous stresses are also assumed to vanish there. These conditions require

$$\left.\begin{aligned} w = 0, \quad \frac{\partial u}{\partial z} = 0, \quad T = 0, \\ \frac{\partial A}{\partial z}\cos\phi = \frac{\partial A}{\partial x}\sin\phi \quad \text{at} \quad z = z_0 \quad \text{and} \quad z = z_0 + 1, \end{aligned}\right\} \tag{2.13}$$

where u and w are the horizontal and vertical velocity components and the scalar flux function A is related to the magnetic field by $\boldsymbol{B} = \boldsymbol{\nabla} \times (A\hat{\boldsymbol{y}})$. We impose periodic conditions for all the variables on the vertical bounding surfaces at $x = 0$ and $x = 2\pi/k$, where k is the wavenumber of the perturbation.

We are then left with a linear boundary-value problem which is separable in x and t; thus we seek solutions of the form $u = \mathrm{Re}\,(\tilde{u}(z)\,\mathrm{e}^{st+\mathrm{i}kx})$, and similarly with the other variables. Dropping the tilde, the eigenvalue problem for the growth rate s is given by

$$\bar{K}\zeta_0(\mathrm{D}^2 - k^2)A - u\cos\phi - w\sin\phi = sA, \tag{2.14}$$

$$\sigma\bar{K}((\mathrm{D}^2 - \tfrac{4}{3}k^2)\,u + \tfrac{1}{3}\mathrm{i}k\mathrm{D}w) - Q\sigma\zeta_0\bar{K}^2(\mathrm{D}^2 - k^2)A\cos\phi - k\mathrm{i}(z_0 f\rho + f^m T) = f^m su, \tag{2.15}$$

$$\sigma\bar{K}((\tfrac{4}{3}\mathrm{D}^2 - k^2)\,w + \tfrac{1}{3}\mathrm{i}k\mathrm{D}u) - Q\sigma\zeta_0\bar{K}^2(\mathrm{D}^2 - k^2)A\sin\phi + (m - z_0 f\mathrm{D})\,\rho - f^m(\mathrm{D} + m/fz_0)\,T = f^m sw, \tag{2.16}$$

$$\bar{K}\gamma f^{-m}(\mathrm{D}^2 - k^2)\,T - z_0 f(\gamma - 1)(\mathrm{D}w + \mathrm{i}ku) - w = sT, \tag{2.17}$$

$$-f^m((\mathrm{D} + m/fz_0)\,w + \mathrm{i}ku) = s\rho, \tag{2.18}$$

where $f = z/z_0$ and $\mathrm{D} = \mathrm{d}/\mathrm{d}z$.

Notice that when $\phi = 0$ we can seek solutions in which w, ρ and T are real while u and A are imaginary, and s is real. The introduction of the $\sin\phi$ terms in the above equations breaks the symmetry and means that this is no longer possible. In general, therefore, the eigenvalue s is complex and all bifurcations are Hopf bifurcations leading to travelling waves.

3. Results

The eigenvalue problem (2.14)–(2.18) was solved by a modification of a package developed by F. Cattaneo (for details see Cattaneo 1984). The complex eigenvalue s is determined in terms of the dimensionless parameters Q, R_a, z_0, m, γ, σ, ζ_0, k and ϕ. Clearly a full exploration of parameter space is impossible! We therefore concentrate on the particular values $z_0 = 0.5$, $m = 1$ (giving a density contrast $\chi = 3$), $\gamma = \frac{5}{3}$,

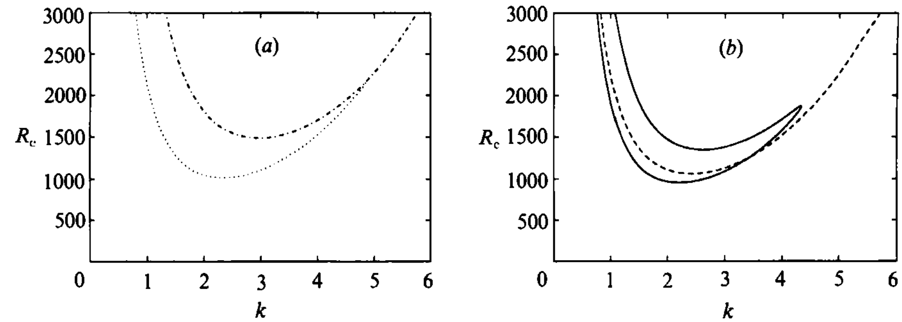

FIGURE 2. The critical Rayleigh number as a function of wavenumber k at $Q = 20$. (*a*) Vertical field: dash-dot line, stationary mode; dotted line, oscillatory mode. (*b*) Field sloping at an angle of $\frac{1}{4}\pi$: solid line, right-going wave; dashed line, left-going wave.

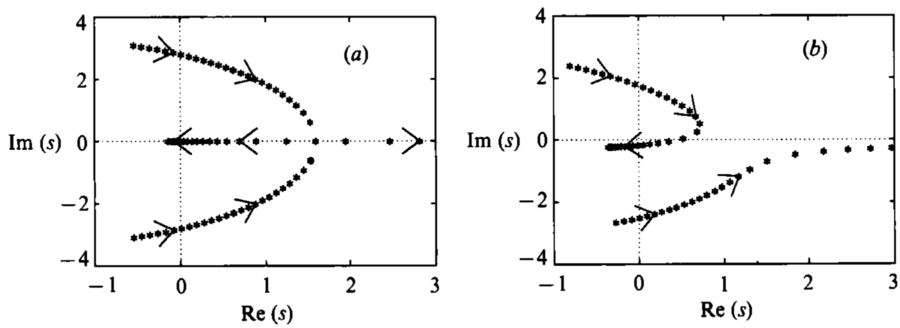

FIGURE 3. The path of the eigenvalues in the complex plane as the Rayleigh number is varied at $Q = 20$, $k = 2.3$. Arrows indicate the direction of increasing Rayleigh number. (*a*) Vertical field. (*b*) Field sloping at an angle of $\frac{1}{4}\pi$.

$\sigma = 1$, $\zeta_0 = 0.05$ and vary R_a, Q, k and ϕ to find bifurcation points at which s is purely imaginary. If $\mathrm{Im}(s)$ is positive (negative) then we have a leftward (rightward) travelling wave, in the sense of figure 1.

The dependence of the critical Rayleigh number R_c (for which $\mathrm{Re}(s) = 0$) on the wavenumber k at $Q = 20$ is shown in figure 2. Figure 2(*a*) shows the case of a vertical field, $\phi = 0$, in which an oscillatory mode becomes unstable at a lower Rayleigh number than the steady mode, provided that k is sufficiently small. Figure 2(*b*) illustrates the symmetry breaking of this oscillatory mode at a field angle $\phi = \frac{1}{4}\pi$.

The path of the eigenvalues in the complex plane as R_a is varied is shown in figure 3, for $Q = 20$ at a fixed wavenumber of $k = 2.3$ which is close to the preferred wavenumber. Figure 3(*a*) shows the symmetric case $\phi = 0$, and figure 3(*b*) shows the situation for $\phi = \frac{1}{4}\pi$ illustrating the symmetry breaking between positive and negative frequencies. This shows that for these parameter values it is a right-going wave which first becomes unstable as the Rayleigh number is increased.

Figure 4 shows the minimum critical Rayleigh number $R_{\min}$ (minimized over k), the wave speed and preferred wavenumber as a function of the angle ϕ for $Q = 1000$ for both the left- and right-going waves. The results show that for most values of ϕ, a right-going wave is preferred. However, for ϕ close to $\frac{1}{2}\pi$, the left-going wave is preferred. The results are qualitatively similar for different values of Q. Values between $Q = 20$ and 10^4 were investigated.

Because the eigenfunctions are complex, the flow is best visualized by multiplying by e^{ikx} and plotting the real part in the (x, z)-plane. Figure 5 shows the flow and the

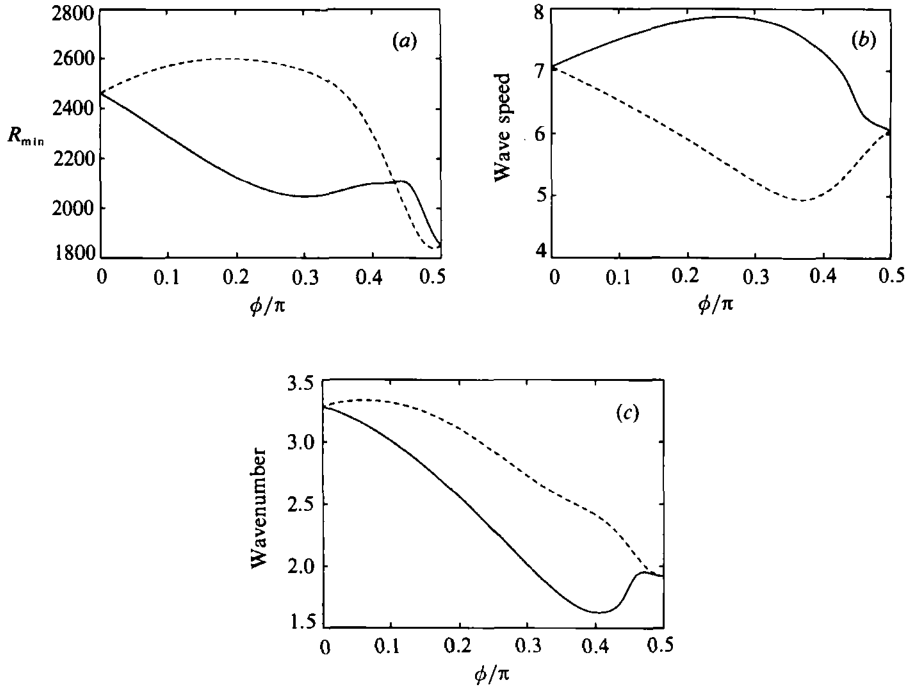

FIGURE 4. (a) Critical Rayleigh number (minimized over k), (b) wave speed, (c) preferred wavenumber, as a function of the field angle ϕ (in units of π), at $Q = 1000$: solid line, right-going wave; dashed line, left-going wave.

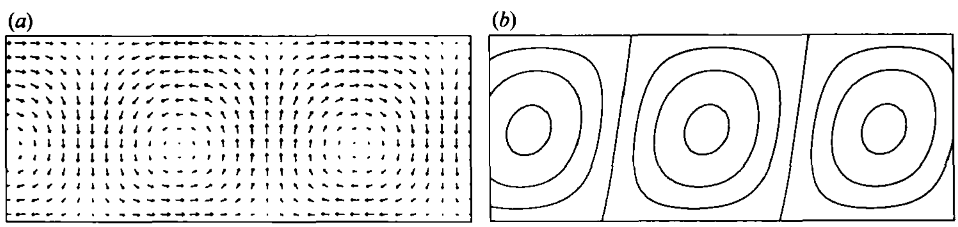

FIGURE 5. Velocity field and temperature perturbation for right-going wave with $\phi = 0$, $Q = 1000$.

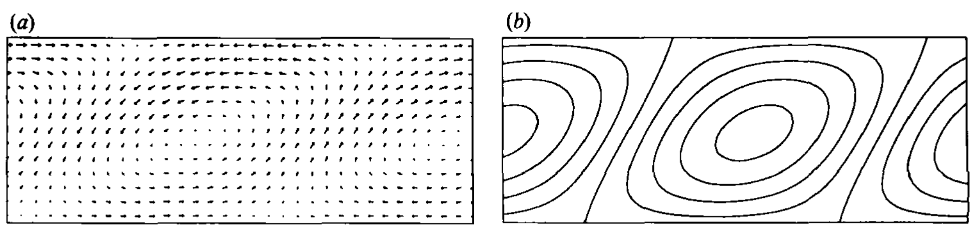

FIGURE 6. As figure 5 but with $\phi = \frac{1}{4}\pi$.

temperature perturbation for the vertical field, $\phi = 0$, for a right-going wave. Notice that although the flow forms rectangular cells, the temperature perturbation forms sloping cells. In the Boussinesq case, there is a constant phase lag with depth between the velocity and the temperature perturbation, but in the compressible case, this phase lag varies with z. The flow and temperature perturbation for $\phi = \frac{1}{4}\pi$ are shown in figure 6. Note that the flow becomes aligned with the magnetic field in this case.

We have also investigated the effect of varying the boundary conditions on the preferred direction of wave travel. For the purposes of a nonlinear calculation, it is necessary to modify slightly the boundary conditions, because the boundary conditions used above result in a horizontal force on the fluid layer which averages to zero in the linear system but causes movement of the entire fluid layer in the nonlinear regime. To avoid this, boundary conditions which remove the Galilean invariance of the problem can be used, for example replacing one of the stress-free boundaries by a no-slip boundary. We have checked that this change does not qualitatively change the results. However, altering the magnetic boundary condition so that the field lines are fixed on the boundaries ($A = 0$) does result in a qualitative change, with left-going waves being preferred for all ϕ. This same behaviour is also found if the magnetic boundary condition of matching to an external potential field is used.

4. The three-dimensional problem

The importance of the two-dimensional oblique-field solutions is greatly enhanced by the fact that for small ϕ these modes are preferred in competition with possible solutions whose wave vector makes a non-zero angle with the direction of tilt. This situation is similar to the case of convection with non-parallel magnetic field and rotation vectors (Eltayeb 1975). When ϕ is small, the problem can be addressed in a very simple way, exploiting the symmetries of the situation and avoiding a detailed analysis of the motion.

Suppose that the direction of tilt is parallel to the x-axis, as shown in figure 1, and let the wave vector of the linearized disturbance be $\boldsymbol{k} = (k\cos\alpha, k\sin\alpha)$.

Consider first the case $\alpha = 0$, corresponding to the two-dimensional problem of the previous section. Then because of the asymmetry between left- and right-going waves, for small ϕ the perturbation to the growth rate (or the minimum critical Rayleigh number) due to the tilt is of order ϕ. Thus in figure 4(*a*) we see that $R_{\min}$ is a linear function of ϕ for sufficiently small ϕ.

Now consider the case $\alpha = \frac{1}{2}\pi$, representing a wave travelling in a direction perpendicular to the plane of tilt. In this configuration the system is symmetric under a sign change of ϕ, since $\boldsymbol{k}$ is unchanged by a reflection in the plane $x = 0$. Thus the perturbation to the growth rate for this mode must be of order ϕ^2. For sufficiently small ϕ therefore, it is clear from symmetry arguments alone that a wave travelling in the plane of tilt ($\alpha = 0$) is preferred over one travelling perpendicular to the tilt ($\alpha = \frac{1}{2}\pi$).

For ϕ of order 1, this argument no longer holds. Indeed for a horizontal field ($\phi = \frac{1}{2}\pi$) we expect that the preferred direction of the wave vector will be $\alpha = \frac{1}{2}\pi$, by analogy with the Boussinesq case.

It can be seen that for $\alpha = \frac{1}{2}\pi$ the situation resembles the vertical field case in that there is no preferred direction to the travelling waves. But any amount of obliquity of α will induce an asymmetry, so the preferred mode will propagate in a definite direction relative to the field tilt. This will remain true even when the field is almost horizontal.

The behaviour for small ϕ can be represented by the following model equation for the perturbation Δs to the growth rate:

$$\Delta s = \phi B(\cos\alpha - 1) + \phi^2 C\sin^2\alpha, \tag{4.1}$$

where $B > 0$ without loss of generality and we choose $C > 0$ to obtain the expected transition to non-zero α. For a fluid which is almost Boussinesq, B is small but C is of order 1. Maximizing Δs over α, we find that the preferred mode is $\alpha = 0$ for $\phi < B/2C$, and $\cos\alpha = B/2C\phi$ for $\phi > B/2C$.

Finally we note that the above discussion does not apply to the travelling wave mode that tends to the steady solution branch as $\phi \to 0$. This mode will have a growth rate that depends on ϕ only through ϕ^2, the sense of tilt appearing only in the direction of travelling. Thus the aligned modes are not necessarily preferred for small ϕ and a detailed solution would be necessary to determine the outcome.

5. Nonlinear development of the travelling waves

The linearized theory presented above shows only the onset of instability. It is clearly of interest to know the fate of travelling waves as their amplitude increases. Extensive studies of the Boussinesq equations with a vertical magnetic field have shown that typically the oscillatory solution branches end on the steady branch (Proctor & Weiss 1982). In the present problem the distinction between steady and travelling wave branches disappears. A proper study would require a full solution of the nonlinear convection problem; however, for illustrative purposes we consider here a simple model that encapsulates the general features and allows some analytical progress. Such a model is readily available as a simple modification of the normal form for the Bogdanov bifurcation with $O(2)$ symmetry (Dangelmayr & Knobloch 1987), which gives a general treatment of the vertical field case for parameters close to those at which the critical Rayleigh numbers for steady and oscillatory modes coincide. The governing equation is fourth order in time and can be written to cubic order in terms of the complex amplitude a as

$$\ddot{a} + \alpha a + \lambda |a|^2 a = \epsilon(\beta \dot{a} - \lambda_2 |a|^2 \dot{a} + \lambda_3 a^2 \dot{\bar{a}}). \tag{5.1}$$

Here α and β are parameters which are related to R_a and Q, and ϵ is a small parameter measuring the distance from the multiple bifurcation point. Steady convection is represented by a steady solution $a = R_0$, while travelling waves are given by $a = R\,e^{i\omega t}$. Because the sign of ω (representing the direction of wave propagation) is immaterial when the field is vertical, all the coefficients in (5.1) are real.

To break the left–right symmetry, we must allow the coefficients in (5.1) to be complex. If the asymmetric terms are $O(\epsilon)$, we may ignore the complex parts of the terms on the right-hand side of (5.1). It is also convenient to discuss the effect of varying a single parameter, r, representing the Rayleigh number. With the magnetoconvection problem in mind, we allow β to increase as r increases, while $\mathrm{Re}(\alpha)$ decreases. Then we obtain the system

$$\ddot{a} + (1-r)\,a + \lambda |a|^2 a = \epsilon(\mathrm{i}c_1 a + \mathrm{i}c_2 |a|^2 a + r\dot{a} - \lambda_2 |a|^2 \dot{a} + \lambda_3 a^2 \dot{\bar{a}}), \tag{5.2}$$

where $-\mathrm{i}c_1$, and $-\mathrm{i}c_2$ are the imaginary parts of α and λ respectively. We can scale the amplitude a so that $\lambda_2 + \lambda_3 = \pm 1$, and we choose the plus sign so as to obtain at least one stable branch of travelling waves. If we seek a neutrally stable travelling-wave solution, $a = R\,e^{i\omega t}$ with ω real, then R and ω are determined by the simultaneous equations

$$-\omega^2 + 1 - r + \lambda R^2 = 0, \tag{5.3}$$

$$c_1 + c_2 R^2 + \omega(r - R^2) = 0. \tag{5.4}$$

In the symmetric case with $c_1 = c_2 = 0$, the travelling waves bifurcate at $r = 0$ and have an amplitude given by $R^2 = r$, and the steady branch bifurcates at $r = 1$ with amplitude $R^2 = (r-1)/\lambda$. The coincident travelling-wave branches intersect the steady branch (and then cease to exist) at $r = 1/(1-\lambda)$, provided $\lambda < 1$.

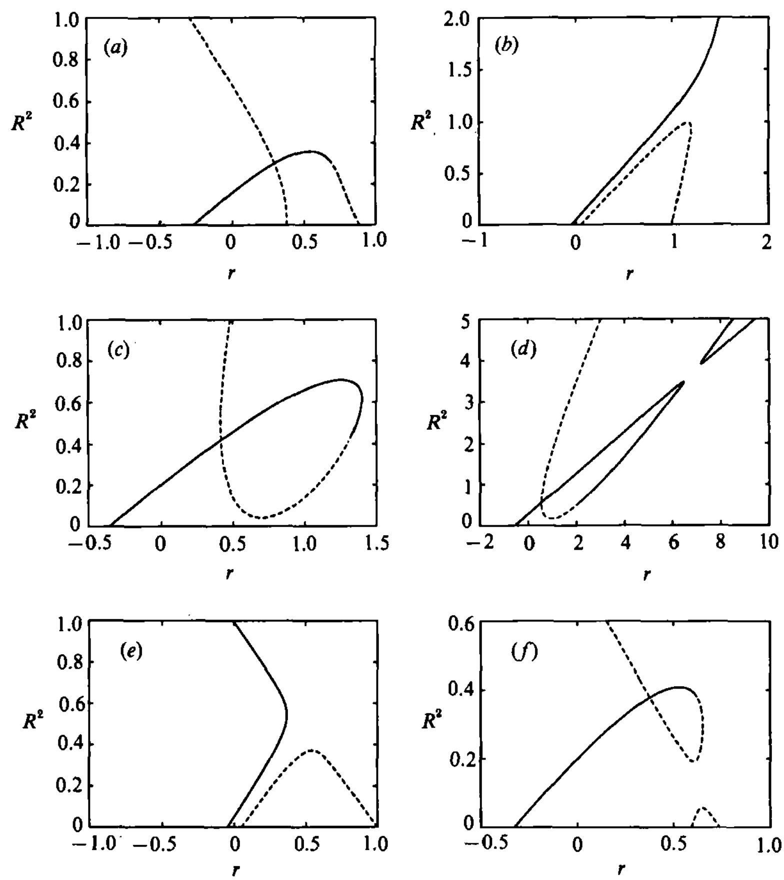

FIGURE 7. Travelling-wave solutions to (5.2): solid line, right-going wave; dashed line, left-going wave. (a) $\lambda = -1$; $c_1 = 0.3$; $c_2 = -1$, (b) $\lambda = 0.25$; $c_1 = 0.05$; $c_2 = 0$, (c) $\lambda = 0.75$; $c_1 = 0.42$; $c_2 = -1$, (d) $\lambda = 2$; $c_1 = 0.7$; $c_2 = -1.25$, (e) $\lambda = -1$; $c_1 = 0.05$; $c_2 = 0$, (f) $\lambda = -1$; $c_1 = 0.378$; $c_2 = -1$.

For the asymmetric system (5.3), (5.4), the bifurcations occur at values of r given by the roots of

$$r^2(1-r) = c_1^2. \tag{5.5}$$

This has three roots if $c_1^2 < 4/27$ and one otherwise, and so is consistent with figure 2, which shows that at a fixed wavenumber there are either three bifurcations or just one. All these bifurcations are Hopf bifurcations, i.e. $\omega \neq 0$.

The nonlinear solutions depend on the parameters c_1, c_2 and λ. We can eliminate ω from (5.3) and (5.4) to obtain a cubic equation for R^2. Without loss of generality we can take $c_1 > 0$. Figure 7 shows some typical examples of the dependence of R^2 on r. For small c_1, c_2 these just show the breaking of symmetry between the two travelling-wave branches. For larger c_1 and c_2, more complicated effects can appear, particularly when $c_1 c_2 < 0$. We have not attempted any detailed analysis of the dependence of the form of the solutions on the various parameters.

The stability of these solutions is complicated by the fact that it depends on the parameters λ_2 and λ_3 separately. However, we can assert that near $R^2 = 0$ the left-most branch is stable if it bifurcates to the right. Some investigations of stability were carried out using the program AUTO. In each of the cases illustrated in figure 7, the left-most branch loses stability at a secondary Hopf bifurcation, giving rise to mixed-mode solutions which are analogous to standing waves in the symmetric case. The position of this bifurcation depends only weakly on the relative sizes of λ_2 and λ_3. These mixed-mode solutions may then terminate on one of the single-mode branches, which subsequently becomes stable, or may end in a homoclinic connection to an unstable branch. Bearing in mind the complexity of the bifurcation diagrams exhibited in Dangelmayr & Knobloch (1987) for the symmetric case, we do not pursue these details here, although they clearly form an interesting subject for future research.

6. Conclusions and discussion

By imposing an oblique magnetic field on a compressible fluid we have broken the symmetries normally associated with convection problems. This means that the eigenvalue problem has complex solutions in general, corresponding to travelling waves. The symmetry breaking means that the oblique-field configuration is more unstable than the vertical-field case with the same total magnetic field strength, at least for small ϕ. When the motion is confined to the plane specified by the tilt of the field, the preferred mode (i.e. that which becomes unstable first as the Rayleigh number is increased) is a wave travelling to the right, in the sense of figure 1, except when the angle of tilt is close to $\frac{1}{2}\pi$. However, this result is sensitive to the precise choice of boundary conditions.

When we consider the problem in three dimensions, the sloping magnetic field resolves the degeneracy of the direction of the wave vector. The symmetry breaking induced by a small angle of tilt ensures that waves travelling in the direction of tilt will be preferred over those travelling perpendicular to the tilt. For larger angles of tilt, however, the angle of the wave vector relative to the plane of the field increases. This symmetry-breaking argument also implies that the preferred wave vector will never be exactly perpendicular to the plane of the field, except when the field is horizontal.

Our nonlinear model for the behaviour of travelling waves shows how the waves may evolve in amplitude as the Rayleigh number is increased beyond critical. The amplitude is given by a cubic equation involving the Rayleigh number and other parameters, so there may be one or three solutions.

This paper raises several questions for future research. It would be of interest to know the nonlinear development of the travelling waves and the possible interactions between them. This problem can be approached either by a numerical solution of the fully nonlinear partial differential equations (Hurlburt, Matthews & Proctor 1992) or by a more thorough investigation of our model equation (5.2). Another important question is whether the symmetry arguments concerning the preferred direction of wave travel relative to the plane of the magnetic field remain valid in the nonlinear regime.

It is interesting to speculate on the relevance of our work to convection in the vicinity of a sunspot. Our results suggest that near the centre of the sunspot, where the field is nearly vertical, waves would travel radially away from the centre. Further away where the field becomes more horizontal, we would expect waves with an almost radial roll axis.

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